

Code No: 151AA

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech I Year I Semester Examinations, June - 2022

MATHEMATICS - I

(Common to CE, ME, ECE, CSE(AI&ML), CSE(IOT))

Time: 3 Hours

Max. Marks: 75

Answer any five questions
All questions carry equal marks

- - -

- 1.a) Apply rank test to find whether the following system has any solution other than $x = y = z = w = 0$, $x + 2y + 3z + 4w = 0$, $8x + 5y + z + 4w = 0$, $5x + 6y + 8z + w = 0$, $8x + 3y + 7z + 2w = 0$.
- b) By using Gauss's elimination method solve $5x - y - 2z = 142$, $x - 3y - z = -30$, $2x - y - 3z = -5$. [8+7]
- 2.a) Find the characteristic polynomial of the matrix $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ and verify Cayley- Hamilton theorem for this matrix. Hence find A^{-1} , if exist.
- b) The matrix $A = \begin{pmatrix} a & h \\ h & b \end{pmatrix}$ is transformed to the diagonal form $D = T^{-1}AT$, where $T = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$. Find the value of θ which gives this diagonal transformation. [8+7]
- 3.a) Prove that the following series for conditional convergence:
- i) $1 - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \dots$ ii) $1 - \frac{1}{2} + \frac{1.3}{2.4} - \frac{1.3.5}{2.4.6} + \dots$
- b) Prove that the series $\frac{\sin x}{1^3} - \frac{\sin 2x}{2^3} + \frac{\sin 3x}{3^3} - \dots$ converges absolutely. [8+7]
- 4.a) Express the $\log(\cos x)$ as Taylor series about $x = \frac{\pi}{3}$.
- b) Find the volume of solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($0 < b < a$) rotated about minor axis. [8+7]
5. If $u = xf(x, y) + yg(x, y)$, show that $\frac{\partial^2 u}{\partial x^2} - 2 \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$. [15]
- 6.a) Determine whether u and v are functionally dependent, where u and v are defined by $u = \sin x + \cos y$ and $v = \cos x + \sin y$.
- b) Find the greatest value of $u = xyz$, if x, y and z are positive real numbers for which $4x + 2y + z = 12$. [8+7]

7. A function $f(x, y)$ is written in terms of new variables $u = e^x \cos y$, and $v = e^x \sin y$. Show that

$$\frac{\partial f}{\partial x} = u \frac{\partial f}{\partial u} + v \frac{\partial f}{\partial v} \quad \text{and} \quad \frac{\partial f}{\partial y} = -v \frac{\partial f}{\partial u} + u \frac{\partial f}{\partial v}, \quad \text{and so deduce that.} \quad [15]$$

8.a) By using techniques involving the Gamma function, find the value of $\int_0^{\infty} \sqrt{x} e^{-\sqrt{x}} dx$.

b) By using techniques involving the Beta function, find the exact value of $\int_0^{\pi/4} \sin^2 2x \cos^4 2x dx$.

[8+7]

--ooOoo--

UNIVERSITY Used Papers June 22