Code No: 181AN **R22**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech I Year I Semester Examinations, January/February - 2024
MATRICES AND CALCULUS
(Common to CE, ME, ECE, EIE, AE, MIE, CSE (AI&ML), CSE(IOT), AI&DS, AI&ML)

Time: 3 Hours **Max. Marks: 60**

Note: This question paper contains two parts A and B.
i) Part- A for 10 marks, ii) Part - B for 50 marks.

- Part-A is a compulsory question which consists of ten sub-questions from all units carrying equal marks.
- Part-B consists of ten questions (numbered from 2 to 11) carrying 10 marks each. From each unit, there are two questions and the student should answer one of them. Hence, the student should answer five questions from Part-B.

PART - A

(10 Marks)

1.a) Write the row elementary transformations on a matrix. [1]
b) State the conditions for the existence of solution of AX=B. [1]
c) Define the Eigenvalues and Eigenvectors of a matrix A. [1]
d) What is meant by index of a quadratic form? [1]
e) Is $f(x) = |x|$ in [-1, 1] differentiable. [1]
f) State Rolle's Mean value theorem. [1]
g) When we say the function is homogeneous? [1]
h) Write the relation between Gamma and Beta functions. [1]
i) Change the order of integral in $\int_0^\infty \int_0^\infty \frac{e^{-y}}{y}\, dy\, dx$. [1]
j) Is $\int_0^1 \int_1^2 \int_2^3 dxdydz = \int_0^1 \int_1^2 \int_2^3 dzdydx$? [1]

PART-B

(50 Marks)

2. Reduce the following matrix $A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 3 & 11 & 6 \end{bmatrix}$ to the (a) Echelon form, (b) Normal form and hence find its rank. [5+5]

OR

3. Using the Gauss-Seidel iteration method, solve the system of equations:
$4x_1 + x_2 + x_3 = 2$, $x_1 + 5x_2 + 2x_3 = -6$, $x_1 + 2x_2 + 3x_3 = -4$. [10]

4. Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence find A^{-1} and
$A^8 - 11A^7 - 4A^6 + A^5 + A^4 - 11A^3 - 3A^2 + 2A + I$. [10]

OR

5. Reduce the quadratic form $x^2+3y^2+3z^2-2yz$ to the canonical form. Hence find its rank, nature, index and signature. [10]

6.a) If $f(x) = \log x$ and $g(x) = x^2$ in $[a, b]$ with $1<a<b$ using Cauchy's mean value theorem, prove or disprove that $\frac{\log b-\log a}{b-a}=\frac{a+b}{2c^2}$ for a < c < b.

b) Evaluate the Taylor series expansion of $f(x)=\tan^{-1}x$ about $x=0$. [4+6]

OR

7. Determine the following

a) $\int_0^1 x^4\left[\ln\left(\frac{1}{x}\right)\right]^3 dx$.

b) $\int_0^{\pi/2}\left(\sqrt{\tan\theta}+\sqrt{\sec\theta}\right)d\theta$. [5+5]

8.a) If $xz+yz-x^2-y^2=0$, then show that $\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)^2=4\left(1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)$.

b) Given that $y_1=\frac{x_2x_3}{x_1}$, $y_2=\frac{x_3x_1}{x_2}$, $y_3=\frac{x_1x_2}{x_3}$. Show that $\frac{\partial(y_1, y_2, y_3)}{\partial(x_1, x_2, x_3)}=4$. [5+5]

OR

9. Discuss the maxima and minima of $x^3y^2(1-x-y)$. [10]

10.a) Compute the double integral $\iint_D (x^2+y^2)\,dx\,dy$, where D is bounded by $y=x$ and $y^2=4x$.

b) Evaluate $\int_0^a\int_y^a \frac{x}{x^2+y^2}dxdy$ by transforming into polar coordinates. [4+6]

OR

11.a) Obtain the value of $\int_0^2\int_1^z\int_0^{yz} x\,y\,z\;dx\,dy\,dz$.

b) Determine the triple integral $\iiint xyz\;dx\,dy\,dz$ taken through the positive octant of the sphere $x^2+y^2+z^2=a^2$, where a is a constant. [3+7]

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