Code No: 181AN **R22**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech I Year I Semester Examinations, March/April - 2023

MATRICES AND CALCULUS

(Common to EEE, CSE, IT, CSIT, CE(SE), CSE(CS), CSE(DS), CSD)

Time: 3 Hours **Max. Marks: 60**

Note: This question paper contains two parts A and B.
i) **Part- A** for 10 marks, ii) **Part - B** for 50 marks.
- Part-A is a compulsory question which consists of ten sub-questions from all units carrying equal marks.
- Part-B consists of **ten questions** (numbered from 2 to 11) **carrying 10 marks each**. From each unit, there are two questions and the student should answer one of them. Hence, the student should answer five questions from Part-B.

PART- A

(10 Marks)

1.a) Are the system of equations $x + y + z = 6,\ x + 2y + 3z = 14,\ x + 4y + 7z = 30$ Consistent? [1]

b) Find the rank of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$. [1]

c) If λ be a eigen value of a matrix A (non-zero matrix). Is λ^{-1} is an eigen value of A^{-1}? [1]

d) The eigenvalues of a matrix $S = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ are 5 and 1. What are the eigenvalues of S^2? [1]

e) Does Lagrange mean value theorem for the function $f(x) = \log_e x$ in the interval [1, e] apply? [1]

f) Write the Beta function β (m, n) in terms of Sine and Cosine. [1]

g) Find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ where $u = e^x \sin y \ \ and \ \ v = x + \log \sin y$. [1]

h) If $u = x^y$, then find $\frac{\partial u}{\partial y}$. [1]

i) Find the area lying between the parabola $y = 4x - x^2$ and the line $y = x$. [1]

j) Change the order of integration for $\int_0^a \int_0^y f(x,y)dx\,dy$. [1]

PART - B

(50 Marks)

2.a) Use either the Gaussian Elimination or the Gauss Jordan method to solve

$x + 2y - 3z = 9$
$2x - y + z = 0$
$4x - y + z = 4$

b) For what values of n will the equations:
$x = y + z = 1;\ \ x + 2y + 4z = n;\ \ x + 4y + 10z = n^2$ be consistent. Solve them completely in each case. [5+5]

OR

3. Using Gauss-seidel method solve the following system of equations
$5x + 2y + z = 12, x + 4y + 2z = 15, x + 2y + 5z = 20.$ [10]

4.a) For matrix $A = \begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$, find non-singular matrices P and Q such that PAQ is in normal form and hence find rank of matrix *A*.

b) Diagonalise the Hermitian matrix $A = \begin{bmatrix} 2 & 1-2i \\ 1+2i & -2 \end{bmatrix}$ to unitarily similar diagonal matrix. [5+5]

OR

5.a) Find a matrix P that transforms $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ to diagonal form and hence find A^4.

b) Using Cayley Hamilton Theorem, find A^8 if $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$. Also find A^{-1}. [5+5]

6.a) State Rolle's Theorem. And Test the validity of the theorem for the functions in the interval mentioned against them $(x-2)^2$ in [-1,5].

b) Show that $\Gamma(n) = \int_0^1 \left(log\frac{1}{x}\right)^{n-1}\ n > 0.$ [5+5]

OR

7.a) Prove that $\beta(n,n) = \dfrac{\sqrt{\pi}.\Gamma(n)}{2^{2n-1}.\Gamma\left(n+\frac{1}{2}\right)}$.

b) The curve $y^2(a+x) = x^2(3a-x)$ revolves about the axis by x. Find the volume generated by the loop. [5+5]

8.a) Determine if the following function is functionally dependent. If they are functionally dependent, then find a functional relation between them.
$u = x\sqrt{1-y^2} + y\sqrt{1-x^2},\ \ v = \sin^{-1}x + \sin^{-1}y$

b) Calculate $\frac{\partial(u,v)}{\partial(r,\theta)}$ if $u = 2axy,\ v = a(x^2-y^2)$ where $x = r\cos\theta,\ y = r\sin\theta$. [5+5]

OR

9.a) Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube.

b) Find the maximum and minimum distance of the point (3, 4, 12) from the sphere $x^2 + y^2 + z^2 = 1$. [5+5]

10.a) Evaluate $\iint \frac{dxdy}{x^4+y^4}$, over the region bounded by $y \geq x^2,\ x \geq 1$.

b) If a denotes the radius of the base, h the altitude of a right circular cone, express the volume as a triple integral and evaluate it. [5+5]

OR

11.a) Find the volume of the solid bounded below by the paraboloid $z = x^2 + y^2$ and above by the plane $z = 2y$.

b) A triangular prism is formed by planes whose equations are $ay = bx, y = 0\ and\ x = a$. Find the volume of the prism between the planes $z = 0$ and the surface $z = c + xy$. [5+5]

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