R22

Code No: 181AN

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech I Year I Semester Examinations, September - 2023
MATRICES AND CALCULUS
(Common to CE, EEE, ME, ECE, CSE, EIE, IT, AE, MIE, CSIT, CE(SE), CSE(CS), CSE(AI&ML), CSE(DS), CSE(IOT), AI&DS, AI&ML,CSD)

Time: 3 Hours **Max. Marks: 60**

Note: This question paper contains two parts A and B.
i) **Part- A** for 10 marks, ii) **Part - B** for 50 marks.
- Part-A is a compulsory question which consists of ten sub-questions from all units carrying equal marks.
- Part-B consists of **ten questions** (numbered from 2 to 11) **carrying 10 marks each**. From each unit, there are two questions and the student should answer one of them. Hence, the student should answer five questions from Part-B.

PART- A

(10 Marks)

1.a) Define rank of a matrix. [1]
b) What do you mean by indirect methods? [1]
c) What is characteristic equation of a matrix? [1]
d) Write the symmetric matrix in the matrix notation of the quadratic form: $2xy + 2yz + 2zx$. [1]
e) State Lagrange's mean value theorem. [1]
f) Define improper integral. [1]
g) State Euler's theorem. [1]
h) What are stationary points of $z = f(x, y)$? [1]
i) Define volume integral of a vector point function $\overline{F}$ over a region V. [1]
j) For finding the area of a plane region R, which double integral is to be evaluated. [1]

PART - B

(50 Marks)

2.a) Determine the rank of the matrix by reducing it into normal form $\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$.

b) Apply Gauss elimination method to solve the equations
$x + 4y - z = -5;\ x + y - 6z = -12;\ 3x - y - z = 4.$ [5+5]

OR

3.a) Apply Gauss -Jordon method to solve the equations:
$2x + 3y - z = -1, x + 4y + 5z = 25, 3x - 4y + z = 2.$
b) Apply Gauss -Siedel method to solve the equations:
$2x + y + 6z = 9,\ 8x + 3y + 2z = 13, x + 5y + z = 7.$ [5+5]

4.a) Find the Eigen values and Eigen vectors of the following matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 1 & -4 & 3 \end{bmatrix}$.

b) Using Cayley-Hamilton theorem the matrix A= $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ compute A^{-1}. Also find for $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$. [5+5]

OR

5.a) Using Cayley Hamilton theorem for the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and find its inverse.

b) Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy - 8yz + 4zx$ into a sum of squares by an orthogonal transformation and also discuss its nature. [5+5]

6.a) Verify Rolle's theorem for the function $f(x) = \log_e x$ in $[1, e]$.

b) Expand $f(x, y) = \sin xy$ in powers of $(x-1)$ and $\left(y - \pi/2\right)$ upto second degree terms. [5+5]

OR

7.a) Find the volume of the revolution of a loop of the curve $y^2(x+b) - x^2(3b-x) = 0$ about x-axis.

b) Prove that $\Gamma(m)\Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$. [5+5]

8.a) If $u = \cos ec^{-1}\left(\frac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}\right)^{1/2}$ then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left[\frac{13}{12} + \frac{\tan^2 u}{12}\right]$.

b) If $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1} x + \tan^{-1} y$, find $\frac{\partial(u,v)}{\partial(x,y)}$. Are u and v functionally related? If so find its relation. [5+5]

OR

9.a) Find the maximum and minimum values of $x^3y^2(1 - x - y)$.

b) A rectangular box open at top is to have a volume of 32 cubic feet. Find the dimensions of the box requiring least material for its construction. [5+5]

10.a) Evaluate $\iint xydxdy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$.

b) Evaluate the integral $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dydx$ by changing the order of integration. [5+5]

OR

11.a) Find by double integration the area of the lemniscate $r^2 = a^2 cos2\theta$.

b) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. [5+5]

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