

Code No: 181AC

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech I Year I Semester Examinations, January/February - 2024

BASIC MATHEMATICS

(Bio-Technology)

Time: 3 Hours

Max. Marks: 60

Note: This question paper contains two parts A and B.

i) **Part - A** for 10 marks, ii) **Part - B** for 50 marks.

- Part-A is a compulsory question which consists of ten sub-questions from all units carrying equal marks.
- Part-B consists of **ten questions** (numbered from 2 to 11) **carrying 10 marks each**. From each unit, there are two questions and the student should answer one of them. Hence, the student should answer five questions from Part-B.

**PART - A****(10 Marks)**

- 1.a) If A and B are two matrices, what is condition that  $A+B=B+A$  to be true. [1]
- b) If  $A = \begin{pmatrix} 1 & -5 & 7 \\ 0 & 7 & 9 \\ 11 & 8 & 9 \end{pmatrix}$ , then find the trace of A. [1]
- c) Find the modulus and principal argument of  $1-i$ . [1]
- d) If  $\bar{a} = 3i - 2j + k$ ,  $\bar{b} = 2i - 4j - 3k$  and  $\bar{c} = -i + 2j + 2k$ , then find  $\bar{a} + \bar{b} - \bar{c}$ . [1]
- e) Express  $\sin hx$  in terms of exponential functions. [1]
- f) Find the sum  $\tan^{-1}(1/3) + \tan^{-1}(1/7) + \tan^{-1}(1/13) + \tan^{-1}(1/21)$ . [1]
- g) Find the domain and range of  $f(x) = \frac{x^2 - 9}{x - 3}$ . [1]
- h) Find the period of  $\sin^2 x$ . [1]
- i) Form a differential equation for y if  $y = Ax^2 + B/x$ . [1]
- j) Four dice are thrown independently Find the probability that exactly two dice show up a three or five. [1]

**PART - B****(50 Marks)**

- 2.a) If  $\alpha$  and  $\beta$  are roots of equation  $x^2 + x + 1 = 0$ , then find the value of  $\frac{1}{\alpha^2 \beta^2} + \alpha^3 + \beta^3$ .

- b) Compute the inverse of the matrix  $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$ . [5+5]

**OR**

3.a) If  $\alpha$  and  $\beta$  are roots of equation  $x^2 + ax + 1 = 0$ . If the equation whose roots are  $-\left(\alpha + \frac{1}{\beta}\right)$  and  $-\left(\frac{1}{\alpha} + \beta\right)$  is  $x^2 - 2ax + k = 0$ , then find  $k$ .

b) Evaluate  $\begin{vmatrix} b^2 - ab & b - c & bc - ac \\ ab - a^2 & a - b & b^2 - ab \\ bc - ac & c - a & ab - a^2 \end{vmatrix}$ . [5+5]

4.a) Prove that  $(a + ib)^{m/n} + (a - ib)^{m/n} = 2(a^2 + b^2)^{m/2n} \cos\{(m/n) \tan^{-1}(b/a)\}$ .

b) Show that the points (1, 5), (2, 4) and (3, 3) are collinear. [5+5]

**OR**

5.a) Prove that  $\frac{(\cos \alpha + i \sin \alpha)^4}{(\sin \beta + i \cos \beta)^5} = \sin(4\alpha + 5\beta) - i \cos(4\alpha + 5\beta)$ .

b) If  $\vec{a}, \vec{b}$ , and  $\vec{c}$  are mutually perpendicular vectors of equal magnitudes, then find the angle between the vectors  $\vec{a}$  and  $\vec{a} + \vec{b} + \vec{c}$ . [5+5]

6.a)  $2y = \left( \cot^{-1} \left( \frac{\sqrt{3} \cos x + \sin x}{\cos x - \sqrt{3} \sin x} \right) \right)^2, x \in \left( 0, \frac{\pi}{2} \right)$ , then find  $\frac{dy}{dx}$  ?

b) Find the partial fractions of  $\frac{1}{x^2(x^2 + 5)}$ . [5+5]

**OR**

7.a) Find the value of  $\sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right)$ .

b) Find the partial fractions of  $\frac{3x^2 + 1}{(x + 3)(x - 1)}$ . [5+5]

8.a) Let the function  $f: [0, 1] \rightarrow \mathbb{R}$  be defined by  $f(x) = \frac{4^x}{4^x + 2}$ . Find the value of  $f\left(\frac{1}{40}\right) + f\left(\frac{2}{40}\right) + f\left(\frac{38}{40}\right) + \dots + f\left(\frac{39}{40}\right)$ .

b) If  $(a + \sqrt{2}b \cos x)(a - \sqrt{2}b \cos y) = a^2 - b^2$ , where  $a > b > 0$ , then find  $\frac{dx}{dy}$  at  $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ . [5+5]

**OR**

9.a) If the function  $f(x) = \begin{cases} k_1(x - \pi)^2 - 1, & x \leq \pi \\ k_2 \cos x, & x > \pi \end{cases}$  is twice differentiable, then find the ordered pair  $(k_1, k_2)$ .

b) If  $f(x) = \frac{2 - x \cos x}{2 + x \cos x}$  and  $g(x) = \log_e x, (x > 0)$  then, find  $\int_{-\pi/4}^{\pi/4} g(f(x)) dx$ . [5+5]

10.a) Solve  $\frac{dy}{dx} = \frac{x(2 \log x + 1)}{\sin y + y \cos y}$ .

b) Solve  $\frac{dy}{dx} = \frac{2}{(x+2y-3)}$ .

[5+5]

**OR**

11.a) If A and B are any two events such that  $P(A) = 2/5, P(A \cap B) = 3/20$ , then find the conditional probability,  $P(A / A' \cup B')$ . A' denotes the complement of A. B' denotes the complement of B.

b) A computer producing factory has only two plants  $T_1$  and  $T_2$ .  $T_1$  produces 20% and plant  $T_2$  produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that  $P(\text{computer turns out to be defective given that it is produced in plant } T_1) = 10 P(\text{computer turns out to be defective given that it is produced in plant } T_2)$ . Where  $P(E)$  denotes the probability of an event E. A computer produced in the factory is randomly selected and it does not turn out to be defective. Then find the probability that it is produced in plant  $T_2$ . [5+5]

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