

Code No: 182AR

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech I Year II Semester Examinations, September - 2023

ORDINARY DIFFERENTIAL EQUATIONS AND VECTOR CALCULUS

(Common to CE, ME, ECE, EIE, AE, MIE, CSE (AI&ML), CSE (IOT), AI&DS, AI&ML)

Time: 3 Hours

Max. Marks: 60

Note: This question paper contains two parts A and B.i) **Part - A** for 10 marks, ii) **Part - B** for 50 marks.

- Part-A is a compulsory question which consists of ten sub-questions from all units carrying equal marks.
- Part-B consists of **ten questions** (numbered from 2 to 11) **carrying 10 marks each**. From each unit, there are two questions and the student should answer one of them. Hence, the student should answer five questions from Part-B.

PART - A**(10 Marks)**

- Define an exact equation. [1]
- Write the standard form of a linear differential equation, in x . [1]
- Find general solution of the differential equation $y'' + y' - 2y = 0$. [1]
- Find the PI of $(D^2 + 4D + 13)y = 2e^{-x}$. [1]
- State first shifting theorem of Laplace transforms. [1]
- State convolution theorem of Laplace transforms. [1]
- Define a scalar point function. [1]
- Define the divergence of a vector point function. [1]
- Define work done by force \vec{F} in moving a particle from A to B along the curve C. [1]
- Define a volume integral. [1]

PART - B**(50 Marks)**

- Solve $(e^x + 1)\cos x dx + e^y \sin x dy = 0$.
- If the air is maintained at 20°C and the temperature of the body cools from 100°C to 80°C in 10 minutes, find the temperature of the body after 20 minutes and when the temperature will be 40°C ? [5+5]

OR

- Solve $(1 + y^2)dx = (\tan^{-1}y - x)dy$.
- Prove that the system of confocal and coaxial parabolas $y^2 = 4a(x + a)$ is self-orthogonal. [5+5]
- Solve $(D^2 + 1)y = \sin x \sin 2x$.
- Solve $(D^2 + 4)y = \tan 2x$ by method of variation parameters. [5+5]

OR

- An LCR circuit with battery emf ' $E \sin pt$ ' is tuned to resonance so that $p^2 = \frac{1}{LC}$. If initially the current i and the charge Q be zero, then show that for small values of $\frac{R}{L}$, the current in the circuit at time t is given by $\frac{E}{2L} t \sin pt$. [10]

- 6.a) Find the Laplace transform of $f(t) = e^{-2t} + 3\sin 2t - 2\cos 3t + t + 100$.
 b) Use convolution theorem to find the inverse Laplace transform of $\frac{1}{(s+1)(s+2)}$. [5+5]

OR

- 7.a) Find the Laplace transform of Dirac delta function.
 b) Using Laplace transform, solve the differential equation $(D^2 - 3D - 4)y = 2e^{-t}$ with $y(0) = 1 = y'(0)$. [4+6]
- 8.a) Find the directional derivative of $\phi = 2xy + z^2$ at $(1, -1, 3)$ in the direction of a vector $\vec{i} + 2\vec{j} + 3\vec{k}$.
 b) If $\vec{r} = xi + yj + zk$, prove that $\text{div}\vec{r} = 3$ and $\text{curl}\vec{r} = 0$. [5+5]

OR

- 9.a) Find the values of a, b , and c so that the vector $\vec{F} = (x + y + az)\vec{i} + (bx + 2y - z)\vec{j} + (-x + cy + 2z)\vec{k}$ be irrotational.
 b) Prove that $\text{Curl}(\vec{F} \times \vec{G}) = (\vec{G} \cdot \nabla)\vec{F} - \vec{G}\text{div}\vec{F} - (\vec{F} \cdot \nabla)\vec{G} + \vec{F}\text{div}\vec{G}$. [5+5]

10. If $\vec{F} = 3x^2y\vec{i} + (x^3 - 2yz^2)\vec{j} + (3z^2 - 2y^2z)\vec{k}$, then
 a) Prove that \vec{F} is conservative.
 b) Find its scalar potential ϕ .
 c) Find the work done in moving a particle under this force field from $(2, 1, 1)$ to $(2, 0, 1)$. [10]

OR

11. Verify Green's theorem in plane for $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the region bounded by the parabolas $y^2 = x$ and $x^2 = y$. [10]

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