

Code No: 182AQ

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech I Year II Semester Examinations, January/February - 2024

MATRICES AND ORDINARY DIFFERENTIAL EQUATIONS

(Common to BT, PCE)

Time: 3 Hours

Max. Marks: 60

**Note:** This question paper contains two parts A and B.i) **Part- A** for 10 marks, ii) **Part - B** for 50 marks.

- Part-A is a compulsory question which consists of ten sub-questions from all units carrying equal marks.
- Part-B consists of **ten questions** (numbered from 2 to 11) **carrying 10 marks each**. From each unit, there are two questions and the student should answer one of them. Hence, the student should answer five questions from Part-B.

**PART - A****(10 Marks)**

- 1.a) Define a Skew-Symmetric matrix. [1]  
 b) Define rank of a matrix. [1]  
 c) If  $\lambda$  is an Eigen value of a square matrix A, then find an Eigen value of  $A^{-1}$  and  $\text{adj } A$ . [1]  
 d) State Cayley-hamilton theorem. [1]  
 e) State Rolle's theorem. [1]  
 f) If  $u = \sin\left(\frac{x}{y}\right)$ , find  $\frac{\partial u}{\partial x}$ . [1]  
 g) Write the condition for a differential equation  $Mdx+Ndy=0$  to be exact. [1]  
 h) State the law of natural growth. [1]  
 i) If the roots of the auxiliary equation are 1, 2, and -2, then write the complementary function of a differential equation  $f(D)y = X$ . [1]  
 j) Find the PI of  $(D^2 - 2D + 1)y = e^x$ . [1]

**PART - B****(50 Marks)**

- 2.a) Find the rank of a matrix  $A = \begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$ .

- b) Solve the system of homogeneous equations:  
 $x + 3y - 2z = 0, 2x - y + 4z = 0, x - 11y + 14z = 0$ . [5+5]

**OR**

- 3.a) Define a Hermitian and a skew-hermitian matrices and give an example in each case.  
 b) Solve the equations  $5x+3y+7z=4, 3x+26y+2z=9, 7x+2y+10z=5$ , using a matrix method. [4+6]

4. Find the Eigen values and Eigen vectors of  $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ . [10]

**OR**

5. Using Cayley - Hamilton theorem for the matrix  $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ , find the inverse of A.

Also express  $B = A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$  as a quadratic polynomial in A. [10]

- 6.a) Verify Rolle's theorem for  $f(x) = \frac{\sin x}{e^x}$  in  $[0, \pi]$ .

- b) If  $u = x \log xy$ , where  $x^3 + y^3 + 3xy = 1$ , find  $\frac{du}{dx}$ . [5+5]

**OR**

- 7.a) Verify Cauchy's mean value theorem for the functions  $\log_e x$  and  $\frac{1}{x}$  in  $[1, e]$ .

- b) A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction. [5+5]

- 8.a) Solve  $xy(1+xy^2) \frac{dy}{dx} = 1$ .

- b) Uranium disintegrates at a rate proportional to the amount then present at any instant. If  $M_1$  and  $M_2$  grams of Uranium are present at times  $T_1$  and  $T_2$  respectively, find the half-life of Uranium. [5+5]

**OR**

- 9.a) Solve  $(1+xy)ydx + (1-xy)xdy = 0$ .

- b) If the temperature of the air is  $30^\circ\text{C}$ , and the substance cools from  $100^\circ\text{C}$  to  $70^\circ\text{C}$  in 15 minutes, find when the temperature will be  $40^\circ\text{C}$ ? [5+5]

10. Solve  $y'' + 4y' + 4y = 3\sin x + 4\cos x$ ,  $y(0) = 1$  and  $y'(0) = 0$ . [10]

**OR**

11. Solve by method of variation of parameters  $\frac{d^2y}{dx^2} + a^2y = \sec ax$ . [10]