Code No: 182AQ **R22**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech I Year II Semester Examinations, September - 2023
MATRICES AND ORDINARY DIFFERENTIAL EQUATIONS
(Common to BT, PCE)

Time: 3 Hours **Max. Marks: 60**

Note: This question paper contains two parts A and B.
i) **Part- A** for 10 marks, ii) **Part - B** for 50 marks.

- Part-A is a compulsory question which consists of ten sub-questions from all units carrying equal marks.
- Part-B consists of **ten questions** (numbered from 2 to 11) **carrying 10 marks each**. From each unit, there are two questions and the student should answer one of them. Hence, the student should answer five questions from Part-B.

PART- A

(10 Marks)

1.a) Define an Orthogonal matrix. [1]
b) Define Normal form of a matrix. [1]
c) If 1, 2, and 3 are the Eigen values of a Matrix A, then find the Trace of A and determinant of A. [1]
d) How do you find the inverse of a matrix by using Cayley-Hamilton theorem? [1]
e) State Cauchy's mean value theorem. [1]
f) Find $\frac{dy}{dx}$ from the implicit function $f(x, y) = 0$. [1]
g) State the linear differential equation of n^{th} order. [1]
h) State Newton's law of cooling. [1]
i) Find the general solution of $(D+1)(D-2)y = 0$. [1]
j) Find the PI of $(D^2+4)y = sin2x$. [1]

PART - B

(50 Marks)

2.a) Define a symmetric and skew symmetric matrices and give an example in each case.
b) Discuss the consistency of the system of equations $AX=B$ and hence solve the following system of equations if it is consistent.
$x_1 - x_2 + x_3 + x_4 = 2;\ x_1 + x_2 - x_3 + x_4 = -4;\ x_1 + x_2 + x_3 - x_4 = 4; x_1 + x_2 + x_3 + x_4 = 0$. [4+6]

OR

3.a) Find the rank of the following matrix by reducing it into Echelon form.

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}.$$

b) Determine the values of λ for which the following system of equations may possess nontrivial solution and hence determine the solution for each permissible value of λ. $3x_1 + x_2 - \lambda x_3 = 0;\ 4x_1 - 2x_2 - 3x_3 = 0;\ 2\lambda x_1 + 4x_2 + \lambda x_3 = 0$. [5+5]

4. Using Cayley - Hamilton theorem for the matrix $A=\begin{bmatrix}1&2&3\\2&4&5\\3&5&6\end{bmatrix}$.

Express $B=A^8-11A^7-4A^6+A^5+A^4-11A^3-3A^2+2A+I$ as a quadratic polynomial in A. Also find the inverse of A. [10]

OR

5. Find a matrix P which transforms the matrix $A=\begin{bmatrix}1&1&3\\1&5&1\\3&1&1\end{bmatrix}$ to the diagonal form. [10]

6. State Lagrange's mean value theorem, and using it prove that $(0<a<b<1)$,

$$\frac{b-a}{1+b^2}<\tan^{-1}b-\tan^{-1}a<\frac{b-a}{1+a^2}.$$

Hence show that $\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\frac{4}{3}<\frac{\pi}{4}+\frac{1}{6}$. [10]

OR

7.a) If $u=\frac{x+y}{1-xy}$ and $v=\tan^{-1}x+\tan^{-1}y$, find $\frac{\partial(u,v)}{\partial(x,y)}$. Are u and v functionally related? If so, find the relationship.

b) Examine the function $f(x,y)=x^3+y^3-3axy$ for maxima and minima. [5+5]

8.a) Solve $(1+y^2)dx=(\tan^{-1}y-x)dy$.

b) The rate at which bacteria multiply is proportional to the instantaneous number present. If the original number doubles in 2 hours, in how many hours will it be triple? [5+5]

OR

9.a) Solve $x\frac{dy}{dx}+y=x^3y^6$.

b) If the temperature of the air is 30^0C, and the substance cools from 100^0C to 80^0C in 10 minutes, find the temperature of the substance after 20 minutes. [5+5]

10. Solve $(D-2)^2y=8(e^{2x}+\sin 2x+x^2)$. [10]

OR

11. Solve by method of variation of parameters $\frac{d^2y}{dx^2}+a^2y=\tan ax$. [10]

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