Code No: 183BR **R22**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech II Year I Semester Examinations, February - 2024
NUMERICAL METHODS AND COMPLEX VARIABLES
(Common to EEE, ECE)

Time: 3 Hours **Max. Marks: 60**

Note: This question paper contains two parts A and B.
i) **Part- A** for 10 marks, ii) **Part - B** for 50 marks.

- Part-A is a compulsory question which consists of ten sub-questions from all units carrying equal marks.
- Part-B consists of **ten questions** (numbered from 2 to 11) **carrying 10 marks each**. From each unit, there are two questions and the student should answer one of them. Hence, the student should answer five questions from Part-B.

PART- A

(10 Marks)

1.a) Find the period of $\cos 3x$. [1]

b) If $f(x) = x^2$ in $-2 < x < 2$, $f(x+4) = f(x)$, then find the coefficient of $\cos\frac{n\pi x}{2}$ in the fourier series expansion of f(x). [1]

c) What is the n^{th} divided differences of a polynomial of the n^{th} degree? [1]

d) Write the condition when the Newton-Raphson method fail while solving f(x)=0.
(i) $f'(x)$ is negative (ii) $f'(x)$ is too large (iii) $f'(x)$ is zero (d) Never fails. [1]

e) While applying Simpson's 3/8th rule, to evaluate $\int_a^b f(x)\,dx$, how many sub intervals should the interval [a,b] to be divided? [1]

f) Write Euler's modified iterative formula. [1]

g) Give an example of a differentiable function but not analytic at a given point. Explain the reason. [1]

h) Describe the region $\text{Im}(3/z) < (1/3)$. Is it bounded or unbounded? [1]

i) How identify a singular point as a pole or isolated singularity from Laurent series expansion? [1]

j) Give an example of essential singularity. [1]

PART - B

(50 Marks)

2.a) Find the Fourier series expansion of the function $f(x) = \pi + x$, $-\pi < x < \pi$. Hence find the sum of the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ...$

b) Find the Fourier cosine integral representation of $f(x) = e^{-2x} + e^{-3x}, x > 0$. [5+5]

OR

3.a) Use the integral $\int_{-\infty}^{\infty} e^{-y^2} dy = \sqrt{\pi}$ to find the Fourier transform of $f(x) = e^{-x^2/2}$.

b) Find the complex form of the Fourier transform of

$$f(x) = \begin{cases} \sin(\pi x) & \text{if } -2 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}.$$

[5+5]

4.a) Find the smallest positive root of the equation that lies between 3 and 4, correct to 3 decimal places, using the method of false position.

b) If y(10) = 35.3, y(15)=32.4, y(20)=29.2, y(25)=26.1, y(30)=23.2 and y(35)=20.5, find y(12) using (i) Newton's forward interpolation formula and (ii) Newton's backward interpolation formula. [5+5]

OR

5.a) Find the Newton-Raphson iterative formula to find the p^{th} root of a positive number N and hence find the cube root of 17.

b) Use Stirling's formula to compute tan 89° 26', given in the following table of values tan *x*.

x	89° 21'	89° 23'	89° 25'	89° 27'	89° 29'
tan *x*	88.14	92.91	98.22	104.17	110.90

[5+5]

6.a) Find the value of $\int_0^{\pi/2} \sqrt{1-0.162\sin^2 x}\, dx$ using Simpson's $1/3^{rd}$ rule.

b) Find the values of *y* at $x = \pm 0.1$, using the Taylor series method of third order with *h* = 0.1, given that $\frac{dy}{dx} = \frac{1}{x+y}, y(0) = 2$. [5+5]

OR

7.a) Compute $\int_4^{5.2} \log_e x\, dx$ using $3/8^{th}$ rule of integration, by dividing the interval of integration into 6 equal sub-intervals.

b) Using Runge-Kutta 4^{th} order method**,** find $y(0.1)$ and $y(0.2)$ for the initial value problem, $\frac{dy}{dx} = y - \frac{2x}{y}, y(0) = 1$ with *h*=0.1. [5+5]

8.a) Find the image of the region $|z+1+i|<1$ under the transformation $w = (3-4i)z + 6 + 2i$

b) Find the bilinear transformation that maps the points ∞, *i*, 0 in *z* - plane into points 0, *i*, ∞ in *w* - plane. Hence, find the fixed points of the map. [5+5]

OR

9.a) If $w = f(z) = u + iv$ is an analytic function of *z*, and $u - v = (x-y)(x^2 + 4xy + y^2)$. Find w in terms of z.

b) If $w = f(z)$ is an analytic function of z; prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\log|f'(z)| = 0$. [5+5]

10.a) State the Cauchy's Residue theorem. Use this theorem to evaluate $\oint_C \frac{(3z+1)^2}{z(z-1)(2z+5)}dz$, where $C:|z|=3$.

b) If $\tan z$ is expanded about $z=\pi/2$ as Laurent series in $0<|z-\frac{\pi}{2}|<\frac{\pi}{2}$, find the principal part and classify the singularity at $z=\pi/2$. [5+5]

OR

11.a) Apply calculus of residues to evaluate $\int_0^{2\pi} \frac{d\theta}{1-2a\sin\theta+a^2}$ where $0<a<1$.

b) Locate the zeros and classify the singularities of (i) $\frac{\tan z}{z}$ (ii) $\frac{\pi\cot(\pi z)}{z^2}$. [5+5]

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