Code No: 154CN **R18**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech II Year II Semester Examinations, April/May - 2023
PROBABILITY THEORY AND STOCHASTIC PROCESSES
(Electronics and Computer Engineering)

Time: 3 Hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, Answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART – A

(25 Marks)

1.a) State Bayes Theorem. [2]
b) What is the condition for a function to be Random Variable? [3]
c) Define Chebyshev's inequality function. [2]
d) State central limit theorem for the case of equal distributions. [3]
e) Classify Random Process. [2]
f) State the properties of Autocorrelation for a Wide Sense Stationary process. [3]
g) Write the expression for Power Spectral Density. [2]
h) Give any two spectral characteristics of system response. [3]
i) Define Noise Equivalent Bandwidth. [2]
j) State the properties of Mutual Information. [3]

PART – B

(50 Marks)

2.a) The number of calls received in a telephone exchange follows a Rayleigh distribution with an average of 20 calls per minute. What is the probability that in one-minute duration? (i) No call is received. (ii) Exactly 5 calls are received. (iii) More than 3 calls are received.
b) Define Rayleigh distribution function and find its mean and variance. [5+5]

OR

3.a) State the classical and Axiomatic definitions of Probability.
b) A continuous random variable X has a PDF $f_X(x) = 3x^2$, $0 \le x \le 1$. Find a and b such that
i) $P\{X \le a\} = P\{X > a\}$ ii) $P\{X > b\} = 0.05$ [5+5]

4.a) Find the marginal density of X if the joint density function of the Random Variable X and Y is given by

$$f_{XY}(X,Y) = \begin{cases} 21xy & for\ 0 \le x \le 1 \\ 0 & otherwise \end{cases}$$

b) Obtain the moment generating function of a uniform distribution and hence find its mean. [5+5]

OR

5.a) Explain the properties of joint distribution function with examples.

b) A random variable X uniformly distributed in the interval (0, π/2). Consider the transformation.
Y = $\cos x$, obtain the pdf of Y. [5+5]

6.a) Briefly introduce the concept of random process and categorize its classifications with one example to each one.

b) Find whether the given stationary random process $X(t) = 10\cos(100t + \theta)$ where θ is a random variable with a uniform probability distribution in the interval (-π,π) is WSS or not. [5+5]

OR

7.a) State and prove the properties of Autocorrelation function of a Random Process.

b) A random process $X(t)$ is defined by $X(t) = Acos\lambda t + Bsin\lambda t, -\infty < t < \infty$ where A and B are independent random variables each of which has the value -2 with probability 1/3 and a value 1 with probability 2/3. Find X(t) is a Wide Sense Stationary Process. [5+5]

8.a) Derive the relation between the cross correlation function and cross power spectral density of a Random Process X(t) and Y(t).

b) A random process X(t) is applied to a network with impulse response $h(t) = u(t)e^{-bt}$ where b>0 is a constant. The cross correlation of X(t) is $R_{xx}(\tau) = u(\tau)e^{-b\tau}$ with the output Y(t). Find the autocorrelation Y(t) and average power in Y(t). [5+5]

OR

9.a) A Gaussian random process X (t) is applied to a stable linear filter. Show that the random process Y(t) developed at the output of the filter is also Gaussian.

b) The input voltage to an RLC series circuit is a stationary random process X(t) with E[X(t)] = 2 and $R_{xx}(\tau) = 4 + \exp(-2|\tau|)$. If the voltage across the capacitor is Y(t), find E[Y(t)]. [5+5]

10.a) Derive the Average Noise Figure of Cascaded Networks.

b) An analog signal band limited to 10 kHz is quantized in 6 levels of a PCM system with probabilities of 1/4, 1/5, 1/5, 1/10, 1/10 and 3/20 respectively. Calculate the entropy and the rate of information. [5+5]

OR

11.a) Describe the Quadrature representation of Narrow Band Noise. Also list out the properties.

b) A source is transmitting six messages with probability 0.30, 0.25, 0.15, 0.12, 0.10 and 0.08 respectively. Find the binary Huffman code and determine its average word length, efficiency and redundancy. [5+5]

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