R18

Code No: 154CN

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech II Year II Semester Examinations, February - 2024

PROBABILITY THEORY AND STOCHASTIC PROCESSES

(Electronics and Computer Engineering)

Time: 3 Hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, Answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART – A

(25 Marks)

1.a) State the conditions for a function to be a Random Variable. [2]
b) Define Probability, Set and Sample Space. [3]
c) Define marginal density functions. [2]
d) Write about linear transformations of Gaussian random variables. [3]
e) What is meant by Wide Sense Stationary Process? [2]
f) Define two joint central moments for two dimensional random variable X and Y. [3]
g) Define the power density spectrum of the response of a system to a random input. [2]
h) Explain how is the autocorrelation function related to the power spectrum? [3]
i) Explain the concept of effective noise temperature. [2]
j) In a communication system, why is it important to characterize and model arbitrary noise sources? [3]

PART – B

(50 Marks)

2.a) Define conditional distribution and density functions and list their properties.
b) A continuous random variable X has a PDF $f_X(x) = 5x^2, 0 \leq x \leq 1$. Find 'a' and 'b' such that: i) $P\{X \leq a\} = P\{X > a\}$ ii) $P\{X > b\} = 0.05$. [5+5]

OR

3.a) Consider the experiment of tossing four fair coins. The random variable X is associated with the number of heads showing on the coin. Compute and sketch the CDF of X.
b) The probability density function of a random variable has the form $f_X(x) = 4e^{-kx}u(x)$, where $u(x)$ is the unit step function. Find the probability that $X > 1$. [5+5]

4.a) Given the function

$$f_{XY}(x,y) = \begin{cases} \beta(x+y)^2 & -2 < x < 2, -3 < y < 3 \\ 0 & elsewhere \end{cases}$$

i) Find a constant 'β' such that this is a valid density function.
ii) Determine the marginal density functions $f_X(x)$ and $f_Y(y)$.
b) Determine the moment generating function about origin of the Poisson distribution. [5+5]

OR

5.a) Define and explain the characteristic function and moment generating functions.

b) A random variable X uniformly distributed in the interval (0, $\pi/2$). Consider the transformation Y = $\sin(x)$, obtain the pdf of Y. [5+5]

6.a) A Random Process $x(t) = Acos(2\pi f_c t)$, where A is a Gaussian Random Variable with zero mean and unity variance, is applied to an ideal integrator, that integrates with respect to 't', over $(0, t)$. Check the output of the integrator for stationarity.

b) Explain covariance and its properties. [5+5]

OR

7.a) State and prove the properties of autocorrelation function of random process.

b) Given $\bar{X} = 6$ and $R_{XX}(t, t + \tau) = 42 + 84\exp(-\tau)$ for a random process X(t). Check Whether the process is Wide Sense Stationary and Ergodic. [5+5]

8.a) Given two random processes with cross-correlation function $R_{xy}(\tau) = e^{-|\tau|}$, calculate the cross-power density spectrum $S_{xy}(f)$.

b) The power spectral density of a stationary random process is given by

$$S_{xx}(\omega) = \begin{cases} A, & -k < \omega < k \\ 0, & otherwise \end{cases}$$

Find the auto correlation function. [5+5]

OR

9.a) Briefly explain the concept of cross power density spectrum.

b) A stationary random process $X(t)$ has autocorrelation $R_{xx}(\tau) = 10 + 5cos(2\tau) + 10e^{-2|\tau|}$. Find the dc and ac powers of $X(t)$. [4+6]

10.a) Calculate the thermal noise voltage (rms) across a 50-ohm resistor at room temperature (300 K). Assume a bandwidth of 1 kHz.

b) Consider a communication system where a transistor introduces arbitrary noise with a power spectral density of -115 dBm/Hz. Determine the total noise power within a 2 MHz bandwidth. [5+5]

OR

11.a) Given a communication system with an effective noise temperature of 300 K and a noise equivalent bandwidth of 20 kHz, calculate the total noise power in the system.

b) A communication receiver consists of three amplifiers with noise figures of 5 dB, 4 dB, and 3 dB, respectively. Calculate the overall average noise figure for the receiver. [5+5]

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