

Answer any five questions
All questions carry equal marks

- 1.a) Test for convergence of the series $u_n = \frac{n^2 - n + 1}{n!}$.
- b) Prove that the series is $\frac{1}{2^3} - \frac{1}{3^3}(1+2) + \frac{1}{4^3}(1+2+3) - \frac{1}{5^3}(1+2+3+4) \dots \dots \infty$ conditionally convergent. [6+9]
- 2.a) Verify Rolle's theorem for $f(x) = \frac{\sin x}{e^x}$ in $(0, \pi)$.
- b) A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction. [6+9]
- 3.a) Find the radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the curve $x^3 + y^3 = 3axy$.
- b) Find the envelope of the family of curves $y = mx + \sqrt{a^2m^2 + b^2}$; m is a parameter. [9+6]
- 4.a) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} (x^2y + y^3) dx dy$.
- b) Evaluate by changing the order of integration $\int_0^\infty \int_x^\infty \frac{e^{-y} dy}{y} dx$. [7+8]
- 5.a) Solve the differential equation $y(xy + e^x)dx - e^x dy = 0$.
- b) Find the Orthogonal Trajectories of the family of curves $x^2 + y^2 = ax$. [8+7]
- 6.a) Solve the differential equation $(D^2 + D + 1)y = x^3$.
- b) Solve the differential equation $(D^2 + 5D + 6)y = e^{-3x}$. [8+7]
- 7.a) Find $L[(t^2 + 1)^2]$.
- b) Find Inverse Laplace transform of $\frac{3s + 7}{(s^2 - 2s - 3)}$. [6+9]
- 8.a) Find a unit normal vector to the surface $x^3 + y^3 + z^3 = 3$ at the point $(1, -2, 1)$.
- b) Applying, Green's theorem evaluate $\oint_C (y - \sin x)dx + \cos x dy$ where C is the plane triangle enclosed by the lines $y = 0, x = \frac{\pi}{2}$ and $y = \frac{2x}{\pi}$. [6+9]