Code No: 51002 **R09**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech I Year Examinations, May - 2018

MATHEMATICS-I

(Common to CE, EEE, ME, ECE, CSE, CHEM, EIE, BME, IT, ETM, MMT, AE, BT, AME, MIE, PTM, MSNT, AGE)

Time: 3 hours **Max. Marks: 75**

Answer any five questions

All questions carry equal marks

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1.a) Test the following series for convergence
$$\frac{1}{2}+\frac{1.3}{2.4}.\frac{1}{2}+\frac{1.3.5}{2.4.6}.\frac{1}{3}+.........to\ \infty.$$

b) Prove that the series $\sum 2^n \sin\left(\frac{x}{3^n}\right)$ converges absolutely for all values of x. [7+8]

2.a) Use Lagrange's mean value theorem, prove that $\frac{b-a}{1+b^2}<\tan^{-1}b-\tan^{-1}a<\frac{b-a}{1+a^2}$ and deduce that $\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\frac{4}{3}<\frac{\pi}{4}+\frac{1}{6}$.

b) If $x=uv$ and $y=\frac{u+v}{u-v}$, find $\frac{\partial(u,v)}{\partial(x,y)}$. [8+7]

3.a) Trace the curve $y^2(a-x)=x^2(a+x)$.

b) Find the co-ordinates of the centre of the curvature at any point of the parabola $y^2=4ax$, and hence find its evolute. [7+8]

4.a) Evaluate $\iint y\,dx\,dy$ over the region R, where R is the region bounded by the parabolas $y^2=4x$ and $x^2=4y$.

b) Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx$. [7+8]

5.a) Solve the differential equation $y\left(xy+e^x\right)dx-e^x dy=0$.

b) Show that the system of confocal conics $1\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1,$ where λ is a parameter, is self orthogonal. [7+8]

6.a) Solve $\left(D^2-2D+1\right)y=x^2e^{3x}-\sin 2x+3$.

b) In an $L-C-R$ circuit, the charge 'q' on a plate of condenser is given by $L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=E\sin\omega t$. The circuit is tuned to resonance so that $\omega^2=\frac{1}{LC}$. If initially the current i and the charge q be zero, show that, for small values of R/L, the current at time t is given by $(Et/2L)\sin\omega t$. [7+8]

7.a) Evaluate $L\left\{te^{2t}\sin 2t\right\}$.

b) Solve the differential equation $\frac{d^2x}{dt^2}+4\frac{dx}{dt}+8x=e^{2t}$, given $x(0)=2$ and $x'(0)=2$ using Laplace transforms. [7+8]

8.a) Find the constants a, b and c if the vector $\bar{f}=(2x+3y+az)\bar{i}+(bx+2y+3z)\bar{j}+(2x+cy+3z)\bar{k}$ is irrotational.

b) Apply Green's theorem to evaluate $\oint_C (2x^2-y^2)dx+(x^2+y^2)dy$, where C is the boundary of the area enclosed by the x-axis and upper half of the circle $x^2+y^2=a^2$. [7+8]

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