Code No: 134SC **RA**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech II Year II Semester Examinations, April - 2018
MATHEMATICS – IV
(Mechanical Engineering (Mechatronics))

Time: 3 Hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

1.a) Find the analytic function whose real part is y. [2]
b) Show that xy^2 cannot be real part of an analytic function. [3]
c) Find the kind of singularity for the function $\frac{1}{\sin z - \cos z}$. [2]
d) Evaluate $\int_C \frac{zdz}{(z+2)}$ where C is $|z| = 3$. [3]
e) Find the fixed points of the transformation $w = \frac{z-1+i}{z+2}$. [2]
f) If $w = \frac{1+iz}{1-iz}$, find the image of $|z| < 1$. [3]
g) Write the Dirichlet's conditions for the existence of Fourier series of a function $f(x)$ in the interval $(\alpha, \alpha + 2\pi)$. [2]
h) State and prove linearity property of Fourier transforms. [3]
i) Classify the second order partial differential equation. [2]
j) Solve by method of separation of variables $2x\frac{\partial z}{\partial x} - 3y\frac{\partial z}{\partial y} = 0$. [3]

PART-B

(50 Marks)

2.a) If $u = e^x[(x^2 - y^2)cosy - 2xysiny]$ is a real part of an analytic function. Find the analytic function.
b) Find the conjugate harmonic function of the harmonic function $r = a(1 + cos\theta)$. [5+5]

OR

3.a) If $f(z) = u + iv$ is an analytic function of z and if $u - v = e^x(cosy - siny)$, find $f(z)$ in terms of z.
b) Show that the function $f(z) = z\bar{z}$ is differentiable but not analytic at $z = 0$. [5+5]

4.a) Evaluate $\int_0^{1+i}(x^2 - iy)dz$ along the path $y = x^2$.
b) Verify Cauchy's theorem for the function $f(z) = 3z^2 + iz - 4$ if c is the square with vertices at $1 \pm i$ and $-1 \pm i$. [5+5]

OR

5.a) State and prove Laurent's theorem.
b) Find the Taylor's series expansion of sinz around $z = \frac{\pi}{2}$. [5+5]

6. Evaluate $\int_0^{2\pi} \frac{sin^2\theta}{a+bcos\theta} d\theta$; (a>b>0). [10]

OR

7. Find the bilinear transform which maps the points $z = 0, -i, -1$ into the points $w = i, 1, 0$. Find the image of the line y = mx under this transformation. [10]

8.a) Obtain the half range cosine series for the function $f(x) = x^2$ when $0 < x < \pi$ and find the sum of the series $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots$

b) Expand the function $f(x) = x^2$ in $[-l, l]$. [5+5]

OR

9.a) State and prove Change of scale property of Fourier transforms.

b) Find Fourier sine transform of $f(x) = \frac{1}{x(x^2+a^2)}$ and hence deduce cosine transform of $\frac{1}{x^2+a^2}$. [5+5]

10. The points of trisection of a string are pulled aside through the same distance on opposite sides of the position of equilibrium and the string is released from rest. Derive an expression for the displacement of the string at subsequent time and show that the mid-point of the string always remains at rest. [10]

OR

11. Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 \le x \le a, t > 0$, subject to the conditions $u(0,t) = u(a,t) = 0$ and $u(x,0) = u_0$. [10]

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