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Code No: 134SC

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech II Year II Semester Examinations, December - 2018
MATHEMATICS – IV
(Mechanical Enginering(Mechatronics))

Time: 3 Hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit.
Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

1.a) If $w = f(z) = z^2, \forall z$, find the values of w which correspond to $z = 2 + i$ [2]
b) Prove that $f(z) = z^2$ is differentiable anywhere and find its derivative. [3]
c) State Cauchy's theorem. [2]
d) Evaluate $\int_C e^z dz$ where C is $|z - i| = 2$ [3]
e) Define Bilinear transformation. [2]
f) Find bilinear transformations whose fixed points are $i, -i$ [3]
g) Write the dirichet conditions of Fourier Series. [2]
h) If $F[f(x)] = F(s)$ then prove that $F[f(ax)] = \frac{1}{|a|} F\left(\frac{s}{a}\right)$ [3]
i) Write one dimensional heat equation. [2]
j) Classify the partial differential equation $4\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$. [3]

PART-B

(50Marks)

2.a) Given that $f(z) = \frac{x^3 - y^3}{x^3 + y^3}$ and $f(0) = 0$, Show that $f(z)$ is discontinuous at $z = 0$.
b) Show that an analytic function with constant modulus is constant. [5+5]

OR

3.a) Show that $f(z) = sinz$ is analytic everywhere in the complex plane and find $f'(z)$.
b) Find analytical function whose real part is $\frac{x}{x^2 + y^2}$. [5+5]

4.a) State and prove Cauchy Integral formula.
b) Evaluate $\int_C \frac{z^2 - 2z - 2}{(z^2+1)^2 z} dz$, where C is $|z - i| = \frac{1}{2}$, Using Cauchy Integral formula. [5+5]

OR

5.a) Obtain Taylor's series expansion of $f(z) = \frac{e^z}{z(z+1)}$ about $z = 2$.
b) Find poles and residue of $\frac{3z+1}{(z+1)(2z-1)}$. [5+5]

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6. Show that $\int_0^{\pi} \frac{d\theta}{a+b\cos\theta} = \frac{\pi}{\sqrt{a^2-b^2}}$, $a > b > 0$. [10]

OR

7. Under the transformation $w = \frac{z-i}{1-iz}$, find the image of the circle in the w plane.
a) $|w| = 1$ b) $|Z| = 1$. [5+5]

8. Find a cosine series expansion $f(x) = x\sin x, 0 < x < \pi$, Hence find
$1 + \frac{2}{1..3} - \frac{2}{3.5} + \frac{2}{5.7} - - - -$ [10]

OR

9.a) Find the Fourier cosine transformation of $f(x) = \begin{cases} 1-x^2 & 0 < x < 1 \\ 0 & otherwise \end{cases}$.

b) Find Fourier Sine transformation of $e^{-|x|}$ and hence evaluate $\int_0^{\infty} \frac{x \sin(\alpha x)}{1+x^2}\, dx$. [5+5]

10. Solve $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial t^2}$ subject to the conditions:
a) u is not finite for $t \to \infty$
b) $u = 0$ for $x=0$ and $x = \pi$ for all t
c) $u = \pi x - x^2$ $for\ t = 0\ in\ (0, \pi)$. [10]

OR

11. A tightly stretched stiring with fixed ends at $x = 0$ and $x = l$ is initially in a position given by $y(x, 0) = u_0 \sin^3(\frac{\pi x}{l})$. If it is released from rest form this position. Find displacement y at any distance x from one end and any time 't'. [10]

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