**Code No: 134SC** **RA**

**JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD**
**B.Tech II Year II Semester Examinations, May - 2019**
**MATHEMATICS – IV**
**(Mechanical Engineering (Mechatronics))**

**Time: 3 Hours** **Max. Marks: 75**

**Note:** This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.



## PART- A

**(25 Marks)**

1.a) Show that $f(z) = xy + iy$ is not analytic [2]
b) Find the analytic function whose real part is -y. [3]
c) Find the kind of singularity for the function $\frac{1}{\sin z - \cos z}$. [2]
d) Evaluate $\oint_C \frac{e^z}{(z+1)^2} dz$, where $C$ is the circle $|z-3| = 3$. [3]
e) Define a bilinear transformation. [2]
f) Find the fixed points of $w = \frac{z-1}{z+1}$. [3]
g) Define a periodic function and state the period of $\sin x + \cos 2x + \frac{1}{3}\sin 3x$. [2]
h) State and prove Fourier integral in complex form. [3]
i) State one dimensional wave equation. [2]
j) Explain the method of separation of variables. [3]

## PART-B

**(50 Marks)**

2.a) If $f(z) = u + iv$ is an analytic function in a region R, prove that the curves $u(x, y) = c_1$, $v(x, y) = c_2$ form two orthogonal families.
b) Show that the function $f(z) = z$ is not analytic at $z = \infty$. [5+5]

**OR**

3.a) If $w = \varphi + i\psi$ represents the complex potential for an electric field and $\psi = 3x^2y - y^3$, find $\varphi$.
b) Show that the function $f(z) = e^{-z^{-4}}$, $z \neq 0$ and $f(0) = 0$ is not analytic at $z = 0$, although Cauchy-Riemann equations are satisfied at this point. [5+5]

4.a) Evaluate $\int_0^{1+i} (x - y + ix^2) dz$ along the real axis from $z = 0$ to $z = 1$ and then along a line parallel to imaginary axis from $z = 1$ to $z = 1 + i$.
b) State and prove Cauchy integral formula. [5+5]

**OR**

5.a) If $0<|z-1|<2$ then express $f(z) = \frac{z}{(z-1)(z-3)}$ in a series of positive and negative powers of (z-1).
b) State and prove Cauchy residue theorem. [5+5]

6. Evaluate $\int_{-\infty}^{\infty} \frac{z^2-z+2}{z^4+10z^2+9} dz$. [10]

**OR**

7.a) Show that every bilinear transformation maps the circles in the $z$ – plane onto the circles in the $w$ – plane.

b) Find the bilinear Transformation which maps the points 1, 0, i to $\infty$, -1 and –I in w-plane. [5+5]

8.a) Obtain the half range cosine series for the function $f(x) = x^2$ when $0 < x < \pi$ and find the sum of the series $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots$

b) Find the half range cosine series for $f(x) = x + x^2 \; in \; (0,2)$. [5+5]

**OR**

9.a) State and prove shifting property of Fourier transform.

b) Find the inverse Fourier sine transform $f(x)$ of $F_s\{p\} = \frac{e^{-ap}}{p}$ and hence deduce $F_s^{-1}\{1/p\}$. [5+5]

10. The ends A and B of a rod 20 cm long have the temperatures at $30^0$C and $80^0$C until steady-state prevails. The temperatures of the ends are changed to $40^0$C and $60^0$C respectively. Find the temperature distribution in the rod at time t. [10]

**OR**

11. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^2(\pi x/l)$. If it is released from rest from this position, find the displacement $y(x,t)$. [10]

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